বিদ্যাসাগর বিশ্ববিদ্যালয়

VIDYASAGAR UNIVERSITY

Question Paper

B.Sc. Honours Examinations 2021

(Under CBCS Pattern)

Semester - III

Subject : PHYSICS

Paper : C 5 - T & P

Full Marks : 60 (Theory - 40 + Practical - 20)

Time : 3 Hours

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.

[MATHEMATICAL PHYSICS-II]

(Theory)

Group - A

Answer any ***three*** of the following questions : 12×3=36

1. (a) Write down 'Dirichlet's condition' for a fourier series.

(b) Find the Fourier series for f (x), if $f\left(x\right)=\begin{cases}-\pi & -\pi<x<o\\ x & o<x<\pi\end{cases}$

Deduce that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+...=\frac{\pi^2}{8}$

(c) Show that the shortest curve joining two points is a straight line. 3+(3+2)+4

2. (a) Solve the following equation by the method of separation of variables.

$$\frac{\partial^2 u}{\partial x\, \partial t} = e^{-\lambda} \cos x$$

(b) Show that $\int_o^{\pi/2} Sin^p\theta\, Cos^q\theta\, d\theta = \dfrac{\Gamma\left(\frac{p+1}{2}\right)\Gamma\left(\frac{q+1}{2}\right)}{2\,\Gamma\left(\frac{p+q+2}{2}\right)}$

(c) Prove that $\int_o^1 x^m \left(\log x\right)^n dx = \dfrac{(-1)^n}{(m+1)^{n+1}}\Gamma(n+1)$

(d) Show that $erf\,(\infty) = 1$ 4+3+3+2

3. (a) Find regular singular points of the differential equation.

$$2x^2 \frac{d^2 y}{dx^2} + 3x \frac{dy}{dx} + \left(x^2 - 4\right) y = 0$$

(b) Express f (x) = $4x^3 + 6x^2 + 7x + 2$ in terms of Legendre Polynomials.

(c) Show that $P_n(-x) = (-1)^n P_n(x)$

(d) Find the value of $\int\limits_{-\infty}^{\infty} e^{-x^2} \left[H_n(x)\right]^2 dx$

(e) State Parseval Identity. 2+3+2+3+2

4. (a) Express f (x) = x as a cosine, half range series in o < x < 2

(b) Determine the solution of one dimensional heat equation

$$\frac{\partial u}{\partial t} = C^2 \frac{\partial^2 u}{\partial x^2}$$

Subject to the boundary conditions $u\,(o,\, t) = 0$, $u\,(l,\, t) = 0$, $(t > o)$ and the initial condition $u\,(x,\, o) = x$, l being the length of the bar.

(c) Apply variational principle to find the equation of one dimensional harmonic oscillator.

(d) Prove that,

$$\int xJ_0^2(x)\,dx = \frac{1}{2}x^2\left[J_0^2(x)+J_1^2(x)\right]+C$$ 3+4+2+3

5. (a) Prove that, $\iiint\limits_V x^{l-1}\,y^{m-1}\,z^{n-1}\,dx\,dy\,dz = \dfrac{\overline{|l}\;\overline{|m}\;\overline{|n}}{\overline{|l+m+n+1}}$

(b) Expand the function f (x) = x sin(x), as a Fourier series in the interval $-\pi \le x \le \pi$.

Hence deduce that $\dfrac{1}{1.3}-\dfrac{1}{3.5}+\dfrac{1}{5.7}-\dfrac{1}{7.9}+\ldots=\dfrac{\pi-2}{4}$

(c) Find the complex form of the Fourier series of

$f(x) = e^{ax} \qquad -r < x < r$ 3+(3+2)+4

6. (a) Prove 'Rodrigue's Formula' of Legendre's Polynomial.

(b) Find the value of $\iiint\limits_R (x+y+z+1)^2\,dx\,dy\,dz,$ where R is defined by

$x \ge o,\ y \ge o,\ z \ge o,\ x+y+z \le 1$

(c) A tightly stretched string with fixed end points $x = o$ and $x = l$ is initially in a position given by $y = y_0 \sin^3\left(\dfrac{\pi x}{l}\right)$. If it is released from rest from this position, find the displacement $y(x, t)$.

Group - B

Answer any ***two*** of the following questions : 2×2=4

7. If $x=\sum\limits_{k=1}^{\infty} a_k \,Sin\, kx$ for $-\pi \le x \le \pi$, then find the value of a_2.

8. Show that $\overline{\left|\frac{1}{2}\right.} = \sqrt{\pi}$

9. Prove that $\lim_{x \to o} \frac{J_n(x)}{x^n} = \frac{1}{2^n \overline{|n+1}} \, (n > -1)$

10. Prove the relation between beta and gamma function.

(Practical : Marks - 20)

1. Answer any ***one*** of the following questions : 20×1=20

   (i) Write the necessary formula.

   (ii) Write the computer code PYTHON only.

   (iii) Print the input and output.

   (iv) Display your result graphically.

(a) (i) Write a python program to find the Inverse of the following matrix.

$$A = \begin{pmatrix} 4 & 5 & 7 \\ 3 & 6 & 2 \\ 4 & 1 & 8 \end{pmatrix}$$

(ii) Write a program to compute the value of R from the five set of data of Ohm's law experiment.

V (Volt)	I (mA)
1.5	2.88
3.1	5.83
4.2	8.15
5.6	10.70
6.8	13.13

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(b) (i) Write a computer program to find the Transpose of the following matrix.

$$A = \begin{pmatrix} 5 & 4 & 2 \\ 3 & 7 & 1 \\ 2 & 6 & 8 \end{pmatrix}$$

(ii) An experiment of spring constant determination is performed and the following information is obtained

Mass (g)	50	100	150	200	250
Displacement (cm)	2	4	6	8	11

Fit a straight line F = kx (Hooke's law formula) and plot your fitted graph on the curve with the data. 20

(c) (i) Write a program to find the solution of the linear system of three equations given below.

$5x_1 + 3x_2 + 9x_3 = 2$

$x_1 + 4x_2 - 3x_3 = 12$

$-2x_1 - 3x_2 + x_3 = -9$

(ii) Write a computer program to generate a parabola and plot it using matplotlib module.

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